INFLUENCE OF UNCERTAINTY IN THE INITIAL DATA ON THE RESULTS OF PLANNING TEMPERATURE MEASUREMENTS

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Results are represented of an analysis of the influence of uncertainty in the initial information about the thermophysical characteristic and boundary conditions of the first kind on the results of optimal planning of temperature measurements in nonstationary thermophysial experiments.

The preliminary selection of the temperature measurement scheme assuring that reliable information about the desired quantities will be obtained should be the absolute element of experimental-computational research in determining the thermophysical characteristics of different structural materials. The effective means of solving this problem is based on optimal planning of the measurement scheme [1]. The measurement plane

$$\xi = \{N, \bar{X}\}, \ \bar{X} = [X_1, X_2, \ldots, X_N]^T, \tag{1}$$

where N is the quantity of thermal sensors whose readings are utilized as initial data for the solution of the inverse problem and X_i, $i = \overline{1,N}$ are coordinates of the thermal sensor arrangement, is introduced in the consideration in such an approach.

The optimal plan is determined from the solution of the extremal problem

$$\xi_0 = \operatorname{Arg} \max \Psi(\xi), \ \xi \in \Xi, \tag{2}$$

where $\Psi(\xi)$ is the quality criterion selected for the experiment, and Ξ is the domain of allowable planes.

The unknown temperature dependences of the material thermophysical characteristics are represented in parametric form for the solution of coefficient inverse problems and the problem is reduced to searching for the parameter vector $\bar{P}$ (see [2]). The dimensionality of this vector is chosen in such a manner that the condition of the residual would be satisfied.

The quality criterion of the experiment for a parametrized inverse problem is constructed on the basis of the information matrix

$$M(\xi) = \{\Phi_{j,k}\}, \ j, k = \overline{1, m}, \tag{3}$$

where

$$\Phi_{j,k} = \frac{1}{N} \sum_{i=1}^{N} \int_0^{\tau_m} \varkappa_i(\tau)\, \Theta_j(X_i, \tau)\, \Theta_k(X_i, \tau)\, d\tau;$$

$\kappa_i(\tau)$, $i = \overline{1,N}$ are functions permitting available information about measurement error to be taken into account, $\Theta_k(X, \tau) = \partial T(X, \tau)/\partial p_k$, $k = \overline{1,m}$ are sensitivity functions. In particular, the square root of the minimal eigennumber $\sqrt{\mu_{min}}$ of the information matrix and its determinant det M can be the quality criteria. The set of allowable plans Ξ is formed on the basis of an analysis of the uniqueness conditions for the solution of the inverse problem under consideration [3, 4] and has the form

$$\Xi = \{(N, \bar{X}) : N \geqslant N_{\min}, \ 0 \leqslant X_i \leqslant b, \ i = \overline{1, N}\}, \tag{4}$$

where N_{min} is the least possible number of thermal sensors for which a unique solution of the inverse problem is assured, and b is the linear dimension of the domain in which the direct problem of heat conduction is determined.

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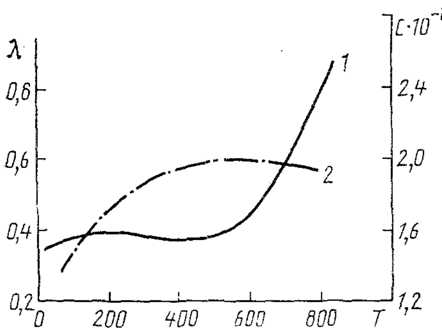

Fig. 1

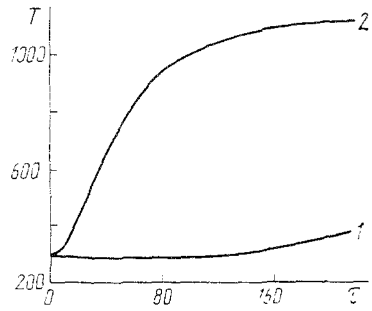

Fig. 2

Fig. 1. Thermophysial characteristics of a material: 1) heat conduction; 2) bulk specific heat; λ, W/(m·deg); J/(m^3·deg); T, °C.

Fig. 2. Temperature change in time on the specimen boundaries: 1) X = 0; 2) b τ sec.

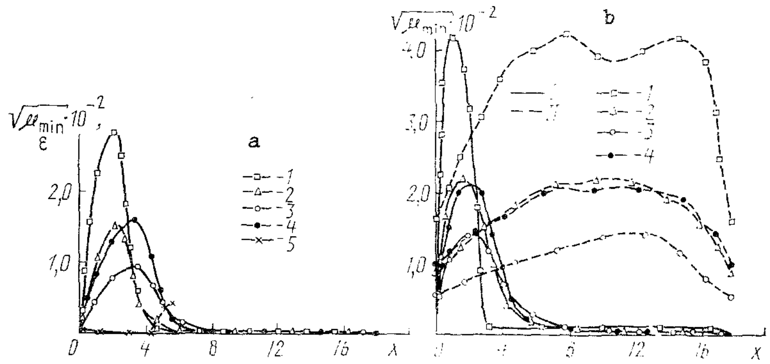

Fig. 3. Influence of uncertainty in giving λ(T) on results of measurement planning: 1) λ = 0.2 W/(m·deg); 2) 0.4, 3) 0.6, 4) λ_{ex} = var, 5) accuracy of inverse problem solution, I, II sections; a) N = 1, b) 2. X, mm.

The temperature T in the heat conduction equation is always nonlinearly dependent on the thermophysial characteristics and therefore, on the parameter vector $\bar{P}$. In this case only locally optimal planning of the measurements is possible with reliance on the a priori information about the desired vector $\bar{P}$ [5].

The solution of the extremal problems (2) to retrieve the locally-optimal temperature measurement schemes requires giving initial data a priori the contain the information needed for the solution of the direct problem, including boundary conditions and identifiable magnetic fields. Investigations carried out earlier show that the optimal planning method using the above-mentioned criteria for accurately known a priori information permits construction of a measuring scheme assuring high accuracy in solving the inverse problem under consideration. However, the preparation and planning of real experiments are carried out under conditions of sufficiently large uncertainty of the initial data. The problem of analyzing the influence of a different kind of uncertainty on the results of oiptimizing a temperature measurement scheme occurs.

The principal uncertainty of the initial information is associated with the need to give a priori data about the characteristics being identified in order to plane the measurements. All available information about the characteristics to be identified should be utilized as initial information. For instance, these can be results of thermophysial measurement obtained earlier. If some data are missing, then estimates of material characteristics can be obtained by knowing its constitution, structure, and the characteristics of individual components [6], and then these data can be utilized to solve the planning problem.

Table 1. Coordinates of the Optimal Thermal Sensor Location during Variation of the Heated Surface Temperature and $\lambda(T) = \lambda_{ex}$

Planning criterion	Number of the boundary mode	N	$\Delta_{max}=0{,}05[g_1(\tau)]_{max}$ X_1^*, mm	X_2^*, mm	$\Delta_{max}=0{,}1[g_1(\tau)]_{max}$ X_1^*, mm	X_2^*, mm
det M	1	1	3,6		3,6	
		2	3,2	11,6	3,2	11,6
	2	1	4,0		4,0	
		2	3,6	11,6	3,6	11,6
	3	1	2,0		3,2	
		2	1,6	11,6	2,0	11,6
	4	1	3,6		3,6	
		2	3,2	11,6	3,2	11,6
	5	1	3,6		2,0	
		2	1,6	11,6	1,6	11,6
$\sqrt{\mu_{min}}$	1	1	3,6		3,6	
		2	2,0	9,6	2,0	9,6
	2	1	2,8		3,2	
		2	2,0	8,0	2,0	8,4
	3	1	3,6		2,8	
		2	1,6	11,2	1,2	11,6
	4	1	3,2		2,8	
		2	1,6	9,6	1,6	9,6
	5	1	3,6		3,6	
		2	1,6	9,2	1,6	9,2

It is ordinarily known in advance to what class the material under investigation belongs when conducting nonstationary thermophysial investigations. On the basis of results of the investigations this permits designating the range of values that the desired thermophysial characteristics can take on. The fundamental question occurring here is the determination of the degree of conformity between the locally optimal measurement plans obtained by using available uncertain information and the exact plane.

The influence of uncertainty in the initial information about the thermophysial characteristics being identified was analyzed in this paper in an example of selecting a rational measuring scheme for an experiment aimed at determining the temperature dependence of the heat conduction coefficient of a glass-plastic in a silicon-organic binder. The material thermophysial characteristics are known and shown in Fig. 1.

For the inverse problem under consideration $N_{min} = 1$ in the relationship (4). The det M and $\sqrt{\mu_{min}}$ were considered as optmality criteria of the measurement plans.

According to available data (see [7]), the range of possible changes in the heat conduction coefficient $\lambda(T)$ of the glass plastic being analyzed is 0.2-0.7 W/(M·K). Three values were selected in this range: 0.2, 0.4, and 0.6 W/(m·K) and locally-optimal temperature measurement plans are constructed for each of them by the methodology in [8]. The temperature distribution at the initial time was here taken constant and equal to 290 K. The given boundary conditions of the first kind are presented in Fig. 2. The specimen thickness b was 17.2 mm.

Results of computations for the construction of locally-optimal plans are represented in the table and in Fig. 3, where the change in one of the planning criteria ($\sqrt{\mu_{min}}$), which corresponds to searching for an exact measurement plane. The results obtained show that locally-optimal measurement plans are sufficiently close to each other for different values of the heat conduction from the range designated. Presented for comparison in Fig. 3 are the results of a parametric analysis of the accuracy of the inverse problem solution in the case of one thermal sensor installation. These results are obtained by using the method from [9]. It is seen that locally-optimal measurement plans assure very high accuracy of solution of the inverse problem.

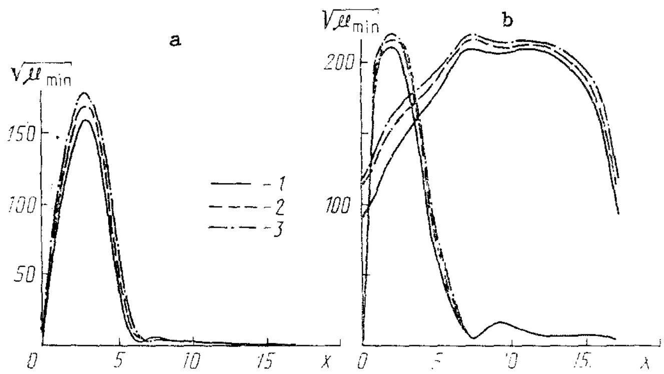

Fig. 4. Influence of the heated surface temperature on the measurement planning results: a) N = 1, b) 2; 1) k = 0; 2) 0.05; 3) 0.1.

The data of the analysis performed indicate that locally-optimal measurement plans in thermophysial investigations of separate classes of materials under nonstationary heating conditions depend sufficiently weakly within the framework of this class on the a priori information about the desired characteristics. These results confirm the possibility of effective application of preliminary locally optimal planning of temperature measurements.

It should be noted that the analysis whose results are elucidated above was performed under the assumption that the boundary conditions of the first kind are known exactly. In practice, information about the boundary conditions is known with a certain error in the majority of cases. The heating mode of the models and specimens under investigation is ordinarily checked by using readings of the heat sensor located most closely to or directly upon the surface being heated. At the same time, readings of the same sensor are used as a boundary condition of the first kind for the determination of the temperature dependence of the heat conduction coefficient. Because of a different kind of errors and deviations from the nominal operating modes of the experimental equipment as well as of the measuring and controlling apparatus, deviations from the required (nominal) law of temperature variation in time inevitable occur at the specimen point selected for checking. Consequently, the need occurs to analyze the influence of possible deviations on the temperature measurement planning results. To this end, computations were carried out on selecting optimal measuring schemems in the case of unilateral heating of specimens during variation of the laws of temperature change in time on the surface being heated.

Under unilateral heating of specimens, the boundary conditions of the first kind cannot be designated independently of each other. These conditions are interrelated by the heat transfer process within the specimen being investigated and the boundary mode on the inner surface. Consequently, the condition of the first kind on the second boundary should be obtained by computational means during arbitrary variation of the temperature mode on the surface being heated. Different computational schemes ca be considered when performing such computations. It was assumed in this paper that the specimen inner surface is heated insulated while a law of temperature change in time is given on the outer uncertainty in giving the heat conduction coefficient.

The computations were performed as follows: a nominal boundary condition was given on the surface being heated, the direct problem was solved, and the temperature change in time was computed on the inner boundary and then used as a boundary condition of the first kind in the planning problem; the measurement plane optimal for the solution of the inverse problem was determined. The temperature mode on the outer boundary was changed after this, the computations were repeated, the optimal plane was determined for the changed mode on the outer boundary, and so forth for all the given variations in the conditions of the first kind on the outer boundary.

Variation of the law of temperature change on the outer surface in time was carried out by two methods. In the first case, systematic deviations growing in time were modelled gradually. The law for the temperature change of the surface being heated with time was calculated from the formula

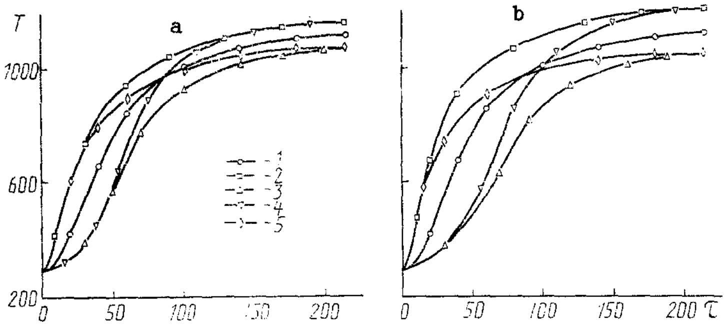

Fig. 5. Variation of the laws of surface temperature change $\Delta = K_2[g_1(\tau)]^{max}_{nom}$. a) K_2 = 0.05; b) 0.1; 1-5) are numbers of the boundary mode, T, K, τ, sec.

$$g_1(\tau) = [g_1(\tau)]_{nom} + K_1\tau/\tau_m,$$

where K_1 is the temperature being variated.

The corresponding changer in the optimality criterion $\Psi = \sqrt{\mu_{min}}$ in the solution of the planning problem for two thermal sensors is represented in Fig. 4. It is seen that the optimal measurement plane remains practically unchanged for the variations under consideration for the law of temperature variation on the external surface.

A certain band of possible temperature deviations from the nominal on the outer surface was designated in the second method. The band width was determined from the condition

$$\Delta = K_2[g_1(\tau)]^{max}_{nom},$$

where K_2 is the parameter being variated. Within the framework of the isolated band different dependences $g_1(\tau)$ were carried out and the optimal temperature measurement plan was determined for each of them according to the scheme described above. Certain considered dependences $g_1(\tau)$ for two values K_2 = 0.05 and K_2 = 0.1 are shown in Figs. 5a and b, while their corresponding solutions of the planning problem are displayed in the Table for two optimality criteria.

The results obtained show that within the limits of the temperature mode deviations from the nominal considered, two thermal sensors are sufficient and the optimal measurement plan remains sufficiently conservative. This confirms the possibility of optimal planning of measurements in both the criteria under consideration under conditions with respect to the uncertain information about the possible thermal boundary conditions of the first kind.

The data of the investigations performed prove, on one hand, the legitimacy of locally-optimal temperature measurement planning under conditions of a sufficiently substantial uncertainty in the initial data and, on the other, permit indication of a path to extend the planning results, that consists in searching for optimal measurement schemes in application to nonstationary thermophysial investigations of individual classes of material in similar specimen heating modes.

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